LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2023**

16/17/18UMT6MC03 – DISCRETE MATHEMATICS

Date: 05-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL questions **(10 x 2 = 20)**

1. Define declarative sentence.
2. What is a quantifier? List some universal quantifiers.
3. Prove that in a lattice if $a \leq b$ then $a \oplus b = b$.
4. How do you justify the consistency of any two given premises?
5. Write the symbolic form of the statement, ‘The crops will be destroyed if there is a flood’.
6. Define semigroup homomorphism.
7. When do you say an element to be idempotent?
8. Explain 2-place predicate with an example.
9. Discuss the conditions for a Boolean expression to be symmetric.
10. What is a complemented lattice?

PART – B

Answer any FIVE questions **(5 x 8 = 40)**

11. Construct the truth table for the following formulae

    (a) $\big(Q \wedge (P \rightarrow Q)\big) \rightarrow P$

    (b) $\urcorner(P \wedge Q) \rightleftarrows (\urcorner P \vee Q)$
12. Check whether the formula $\big(Q \wedge (P \rightarrow Q)\big) \rightarrow P$ is a tautology.
13. Prove that the conclusion R ∨ S follows from the premises (C ∨ D) → ˥H, ˥H → (A ∧ ˥B) and (A ∧ ˥B) → (R ∨ S) using equivalence laws.
14. Show that in a complemented distributive lattice $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$.
15. Let $(S,*)$ be a semigroup. Then show that there is a homomorphism $g: S \rightarrow S^S$ when $(S^S,\circ)$ is the semigroup of functions from S to S under the operation of composition.
16. Write the properties that satisfies Boolean algebra with their identities.
17. State and prove De Morgan’s law of Lattice.
18. Define least upper bound and greatest lower bound and prove that every finite lattice is bounded.

PART – C

Answer any TWO questions **(2 x 20 = 40)**

19. (a) Express the following Boolean expressions in an equivalent sum of the product of canonical forms in three variables x_1, x_2 and x_3.
(i) $x_1 * x_2$. (ii) $x_1 \oplus x_2$. $(iii) (x_1 \oplus x_2)' * x_3$.
(b) Obtain the principal disjunctive and conjunctive normal forms of the statement
$(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$ **(10 + 10)**

20. (a) State and prove Stone representation theorem.
(b) Prove that for any commutative monoid $(M, *)$, the set of idempotent elements of M forms a submonoid. **(10 + 10)**

21. (a) Let X be a set containing n elements and X^* denote the free semigroup generated by X, and $(S, \oplus)$ be any other semigroup generated by any n generators. Show that there exists a homomorphism $g: X^* \rightarrow S$.
(b) Show that $P \rightarrow Q$, $P \rightarrow R$, $Q \rightarrow \neg R$ and P are inconsistent. **(10 + 10)**

22. (a) Prove that the set of all divisors of 24 under the relation of division is a lattice.
(b) Verify using rules of inference whether $S \vee R$ is tautologically implied by
$(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$. **(10 + 10)**

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